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Propagation of the third sound wave in fluid: hypothesis and theoretical foundation☆

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Abstract

A comprehensive account of the emerging concept of dispersion of heat along the axial direction as a fluid flows through a passage bounded by solid wall has been presented with its most recent and remarkable advancement. This new proposition takes axial dispersion as a disturbance which propagates as a wave with a finite velocity. It has been proposed that this sound like propagation be named as the "third sound wave in flowing fluid". The fundamental analysis of this theory has been presented with particular emphasis on the boundary condition which plays a key role in the propagation of the wave. A general flux formulation has been used for this purpose. Analysis has also been presented for a two fluid situation. It has been found that the 'subsonic' and 'super sonic' flow with respect to third sound wave behave differently particularly at entry and exit. The theoretical background developed has been substantiated by three examples—one purely theoretical condition, one comparison with numerical analysis and finally application to a complete apparatus.

Nomenclature

A parameter, equation (44)
a fluid diffusivity based on dispersion $= \frac{\lambda}{\rho c_p}$ [$m^2\ s^{-1}$]
a^* a fluid diffusivity based on molecular conduction, [$m^2\ s^{-1}$]
a_w thermal diffusivity of the wall
B_1, B_2 breadth of the flow channels, Fig. 5 [m]
B' parameter $= \frac{M_w c_w}{m_f c_p}$
C propagation velocity of third sound wave [$m\ s^{-1}$]
c_p specific isobaric heat capacity of the fluid [$J\ kg^{-1}\ K^{-1}$]
C_w second sound velocity in wall [$m\ s^{-1}$]
c_w specific heat of the solid wall [$J\ kg^{-1}\ K^{-1}$]
D parameter, equation (44)
F area
$F(s)$ Laplace transform of the input temperature function
h specific enthalpy [$J\ kg^{-1}$]
H the operator $\left(1 + \frac{a}{C^2}\frac{D}{D\tau}\right)$
L length of the apparatus [m]
M third sound mach number, $\frac{w}{C}$
$\dot{m}$ fluid flow rate [$kg\ s^{-1}$]
m_f mass of hold up fluid in the apparatus [kg]
NTU number of transfer units $= \frac{\alpha F}{\dot{m} c_p}$
Pe dispersive Péclet number $= \frac{w \cdot L}{a} = \frac{\dot{m} c_p L}{\lambda F_q}$
Pe_L conductive Péclet number of the fluid $= \frac{wL}{a^*} = \frac{\dot{m} c_p L}{\lambda^* \cdot F_q}$
$\dot{q}$ heat flux [$W\ m^{-2}$]
$\dot{q}_x$ dispersive axial heat flux [$W\ m^{-2}$]
$\dot{q}_s$ convective heat flux from (or to) the wall [$W\ m^{-2}$]
R tube radius [m]
s transformed time in frequency domain

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T temperature [K]
ΔT temperature rise (or drop) [K]
ΔT_m mean temperature difference [K]
ΔT_{ad} rise of adiabatic mixing temperature [K]
U perimeter [m]
w fluid velocity [m s^{-1}]
x spatial coordinate, flow length [m]
x_0 parameter, equation (48)
y_0 parameter, equation (48)
z dimensionless time $= \frac{\tau w}{L}$.

Greek symbols
α convective heat transfer coefficient [W m^{-2} K^{-1}]
γ_w conduction parameter for wall $= \frac{\lambda}{L\rho_w c_p w}$
δ_s thickness of the plates [m]
ξ dimensionless space coordinate $= x/L$
η slenderness of the tube, R/L
ϑ dimensionless temperature $= \frac{T - T_{at}}{T_{in} - T_{at}}$
λ axial dispersion coefficient [W m^{-1} K^{-1}]
λ^* thermal conductivity [W m^{-1} K^{-1}]
ρ density [kg m^{-3}]
τ time [s].

Subscripts
ad adiabatic
at initial (atmospheric)
c cold
h hot
in inlet
out outlet
q cross-section
s surface
w wall
x in the axial (x) direction
1 channel 1
2 channel 2.

Superscripts
$+$ just after
$-$ just before
$*$ molecular value
$(\bar{\ })$ Laplace transformed.

1. Introduction

Both steady state and transient response of heat exchangers have received considerable attention during the last decade. On one side the steady state analyses [1–3] have stressed the need for theoretical and experimental evaluation of heat exchanger performance during normal operation. On the other hand the transient studies [4–8] have brought out the response features due to off-normal behaviour which are of immense importance for imparting proper control strategy [9]. Even though the advent of high speed computing systems has made the complete numerical simulation of heat exchangers possible, still the need for simpler and more accurate simulation of temperature response remains equally important because the numerical simulations are too complex to be directly used for the purpose of design or control of heat exchangers. A significant breakthrough in this direction has been achieved using the concept of apparent axial dispersion phenomenon in mass transfer during turbulent flow [10]. From heat and mass transfer analogy it is established [11, 12] that the same concept can be used for heat transfer as well. The studies [1–8] attribute all the contribution for the deviation from conventional plug flow such as backmixing, recirculation, leakage, bypassing, flow maldistribution and stagnation to an axial heat dispersion. The essence of this dispersion model is plug flow with an apparent diffusive flux propagation through the fluid due to the above deviations from normal plug flow which can be represented by a conduction like diffusive flux equation, as

$$\dot{q} = -\lambda \nabla T. \tag{1}$$

The major difference between this apparent conductivity λ and the real conduction in fluid is that the apparent conductivity λ is a function of macroscopic flow parameters because its origin lies also in macroscopic non-idealities. On the other hand fluid conduction is a molecular phenomenon and is a microscopic property of fluid itself.

The apparent conduction equation (1) based on the Fourier law assumes an infinite propagation velocity of the thermal wave which makes it diffusive in nature to result in a parabolic temperature equation,

$$\frac{\partial T}{\partial \tau} = a^* \frac{\partial^2 T}{\partial x^2}. \tag{2}$$

Chester [13] proposed a 'non-Fourier' (also known as 'hyperbolic' or 'wave') model of conduction which was found to be of practical importance at near absolute zero temperature. This formulation assumes a heat flux in the form

$$\dot{q} + \frac{a^*}{C^{*2}} \frac{\partial \dot{q}}{\partial \tau} = -\lambda^* \nabla T \tag{3}$$

resulting in a hyperbolic temperature equation for one-dimensional transient conduction

$$\frac{\partial T}{\partial \tau} + \frac{a^*}{C^{*2}} \frac{\partial^2 T}{\partial \tau^2} = a^* \frac{\partial^2 T}{\partial x^2}. \tag{4}$$

It is important to note that the relaxation time given by a^*/C^{*2} approaches zero as the velocity of propagation for thermal wave approaches infinite reducing the hyperbolic conduction equation (4) to the conventional parabolic one [equation (2)]. However, it seems to be logical to differentiate the molecular conduction from axial dispersion from the viewpoint of the propagation velocity. The axial dispersion is a physical disturbance which propagates through the fluid and hence cannot propagate at infinite speed.

An extension of the conventional axial dispersion model for heat exchangers taking into account a finite propagation velocity of the axial dispersion by incorporating Chester's constitutive equation (3) applied to a fluid flowing with a given mean velocity was first proposed by Roetzel and Spang [14, 15]. This model contains as a second parameter the finite propagation velocity which may be termed as the 'third sound wave in a flowing fluid' (second being the molecular conduction).

Shortly after Roetzel and Spang [14, 15] and independently of them Westerterp *et al.* [16, 17] developed and analysed a similar model for longitudinal mass dispersion in chemical reactors. Based on an elaboration of dispersion of a solute in laminar flow following Taylor's approach [18] they arrived at a more general wave model. Compared to Chester's equation (3) their constitutive equation for the dispersion flux (in their case mass dispersion) additionally contains the derivative of the source term (in their case chemical reactions) with respect to the driving force (in their case concentration) and a third parameter for velocity asymmetry. Westerterp and coworkers subsequently applied their wave model to tubular reactors [19, 20].

Further work has also been done in applying the wave model to heat exchangers [21, 22]. The wave like propagation of dispersion with finite velocity was based on a discrete study in a plate exchanger without paying much attention to the boundary conditions [21] which appears to be of immense importance. Also it was presented from a purely logical approach without any direct proof validating such behaviour.

The present study brings out a more comprehensive background for the wave model of the propagation of dispersion in the form of a third sound wave along with the 'subsonic' and 'supersonic' behaviour at boundary. Different examples are presented to validate and reiterate the necessity of taking the finite propagation velocity of the dispersion wave into consideration which is established with physical reasoning. The concept of a third sound will not only revolutionise the study of thermal equipment but will also help to look at all thermal hydraulic phenomena with a new point of view. It is important to mention here that the two predominant ways of analysis of thermal hydraulic phenomena are (i) experimental, which in many cases are too empirical and difficult to theorise and (ii) numerical, which is too theoretical due to a stream of assumptions associated with it. The present development can act as a bridge between the two approaches. The parameters associated with third sound wave can be determined from experiment and simple analysis can be carried out to put the experimental findings into proper theoretical perspective.

1.1. *The fundamentals of the 'third sound wave'*

To derive the fundamental equations for this third sound wave in the form of finite propagation velocity of dispersion wave, a stationary horizontal column of fluid in a channel is considered as shown in Fig. 1. First the second sound wave in the form of heat conduction (since the fluid is stationary) is taken into account. With constant properties a one-dimensional analysis with lateral heat transfer from wall can be carried out by energy balance over the elemental fluid volume as shown in Fig. 1 to give

$$\rho c_p \frac{\partial T}{\partial \tau} = -\frac{\partial \dot{q}_x}{\partial x} + \frac{U}{F_q}\dot{q}_s. \quad (5)$$

With the fluid flowing though the tube this equation can be transformed to the third sound wave equation due to dispersion neglecting the molecular conduction in fluid which is of much lower order than dispersion. This can be effected first by replacing the partial time derivative by the substantial differential operator

$$\frac{D}{D\tau} = \frac{\partial}{\partial \tau} + w\frac{\partial}{\partial x} \quad (6)$$

where a constant axial velocity w is assumed for the fluid. Multiplying the transformed equation (5) with the hyperbolic operator [22]

$$H = \left(1 + \frac{a}{C^2}\cdot\frac{D}{D\tau}\right) \quad (7)$$

and utilizing equation (3), the original equation (5) can be written as [21],

$$\frac{1}{C^2}\left[\frac{\partial^2 T}{\partial \tau^2} + 2w\frac{\partial^2 T}{\partial \tau\,\partial x} + w^2\frac{\partial^2 T}{\partial x^2}\right] + \frac{1}{a}\left[\frac{\partial T}{\partial \tau} + w\frac{\partial T}{\partial x}\right] = \frac{\partial^2 T}{\partial x^2} + \frac{U}{\lambda F_q}\left[\dot{q}_s + \frac{a}{C^2}\frac{D\dot{q}_s}{D\tau}\right]. \quad (8)$$

The conduction related quantities like λ, a and C are due to the apparent conduction or the axial dispersion in fluid. The convective flux $\dot{q}_s$ given in this equation could

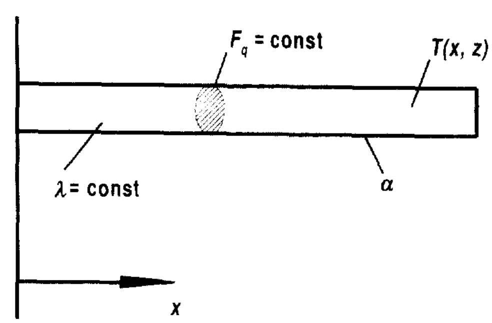

Fig. 1. Schematic of fluid column used for analysis.

also be modelled as a hyperbolic flux which physically introduces a delay period near the wall in the form

$$\dot{q}_s + \frac{a_s}{C_s^2}\frac{D\dot{q}_s}{D\tau} = \alpha(T_w - T). \quad (9)$$

This not only makes the analysis complex but brings out a new definition of convective heat transfer coefficient through equation (9). As such determination of heat transfer coefficient is one of the most complex problems of heat transfer, changing its definition will lead to a situation where no available data of century old studies can be useful. Moreover, since the delay at the wall comes out of the molecular conduction in the adjacent 'no slip' layer of fluid its magnitude is much less than the dispersive delay. Hence this can be neglected and the usual definition of convective heat flux can be used in the form

$$\dot{q}_s = \alpha(T_w - T). \quad (10)$$

This results in a fluid equation of

$$\frac{1}{C^2}\left[\frac{\partial^2 T}{\partial \tau^2} + 2w\frac{\partial^2 T}{\partial \tau\,\partial x} + w^2\frac{\partial^2 T}{\partial x^2}\right] - \frac{\alpha U}{\rho c_p F_q C^2}\left[\frac{\partial (T_w - T)}{\partial \tau} + w\frac{\partial (T_w - T)}{\partial x}\right] + \frac{1}{a}\left(\frac{\partial T}{\partial \tau} + w\frac{\partial T}{\partial x}\right) = \frac{\partial^2 T}{\partial x^2} + \sum \frac{\alpha U}{\lambda F_q}(T_w - T). \quad (11)$$

The summation sign Σ is introduced in front of the heat transfer term because the fluid can wet more than one wall, e.g. the shell and the tubes in a shell-and-tube heat exchanger.

The axial diffusive flux in the form of conduction can be used to model solid wall temperature where the equation (5) holds true and if a hyperbolic model is used in the form of equation (3), it results

$$\rho_w c_{pw} H_w \frac{\partial T_w}{\partial \tau} = \lambda_w \frac{\partial^2 T_w}{\partial x^2} + \frac{U}{F_w} H_w \dot{q}_s \quad (12)$$

where H_w is the hyperbolic wall operator [22] given by

$$H_w = \left(1 + \frac{a_w}{C_w^2}\frac{\partial}{\partial \tau}\right). \quad (13)$$

However, this is of practical importance only at extremely low temperature or very fast transient processes such as impulse heating of fluid where the time scale matches with the relaxation time a_w/C_w^2.

In normal operations of relatively higher temperature and larger time scale the wall conduction propagation velocity C_w can be regarded infinite giving the known wall equation of

$$\rho_w c_{pw} \frac{\partial T_w}{\partial \tau} = \lambda_w \frac{\partial^2 T_w}{\partial x^2} + \sum \frac{U\alpha}{F_{qw}}(T - T_w). \quad (14)$$

The summation sign Σ in equation (14) indicates that the wall can simultaneously be wetted by more than one fluid.

Equations (11) and (14) are the coupled wall and fluid equations which can be used for any heat transfer apparatus carrying fluids in conduits or channels and exchanging heat with the solid walls in contact. Thermal regenerators, recuperative heat exchangers consisting of tubes, plates or fins are the most important examples. A more detailed set of equations taking time delays for heat transfer and wall heat conduction into account is given in [22]. However, equations (11) and (14) are sufficient in all practical cases.

1.2. The dispersive Péclet and Mach numbers

The equations (11) and (14) can be non-dimensionalized by introducing a dispersive Péclet number

$$Pe = \frac{w \cdot L}{a} \quad (15)$$

and a dispersive mach number

$$M = \frac{w}{C}. \quad (16)$$

Since the dispersive wave propagates as a third sound wave it is reasonable to use the term mach number for it.

With these and other normal dimensionless parameters the two equations (11) and (14) reduce to:

$$\frac{M^2}{Pe}\left(\frac{\partial^2 \vartheta}{\partial z^2} + 2\frac{\partial^2 \vartheta}{\partial z\,\partial \xi} + \frac{\partial^2 \vartheta}{\partial \xi^2}\right) - \frac{M^2 NTU}{Pe} \times \left[\frac{\partial}{\partial z}(\vartheta_w - \vartheta) + \frac{\partial}{\partial \xi}(\vartheta_w - \vartheta)\right] + \frac{\partial \vartheta}{\partial z} + \frac{\partial \vartheta}{\partial \xi} = \frac{1}{Pe}\frac{\partial^2 \vartheta}{\partial \xi^2} + \sum NTU(\vartheta_w - \vartheta) \quad (17)$$

$$\frac{\partial \vartheta_w}{\partial z} = \gamma_w \frac{\partial^2 \vartheta_w}{\partial \xi^2} + \sum \frac{NTU}{B'}(\vartheta - \vartheta_w). \quad (18)$$

For steady state situations equation (17) reduces to

$$\frac{M^2 - 1}{Pe}\frac{\partial^2 \vartheta}{\partial \xi^2} + \left(1 + \frac{M^2 NTU}{Pe}\right)\frac{\partial \vartheta}{\partial \xi} - \frac{M^2 NTU}{Pe}\frac{\partial \vartheta_w}{\partial \xi} = \sum NTU(\vartheta_w - \vartheta) \quad (19)$$

and equation (18) to

$$O = \gamma_w \cdot \frac{\partial^2 \vartheta_w}{\partial \xi^2} + \sum \frac{NTU}{B'} \cdot (\vartheta - \vartheta_w). \quad (20)$$

1.3. The boundary conditions

The boundary conditions with third sound wave propagation is rather complex. A unique equation cannot be derived for the whole flow regime as in parabolic dispersion popularly known as the Danckwerts [23] boundary condition. Considering the propagation velocity of third sound wave in the fluid, the flow regime can be divided into subsonic, sonic or supersonic regimes according to the conditions $M < 1$, $M = 1$ or $M > 1$, respectively.

The behavioural difference at subsonic and supersonic flow with respect to dispersive mach number results from the relative importance of the causes of dispersion. The subsonic condition ($M < 1$) is valid for real conduction, mass diffusion or axial mixing known as backmixing. The supersonic condition ($M > 1$) is of importance when it is not a real but a virtual mixing process such as maldistribution which is likely to travel with a propagation velocity below or of the order of fluid velocity.

1.4. Case 1, M < 1

This is the boundary condition used so far in the literature [21]. This can be looked upon as an extension of Dankwerts [23] boundary condition. According to this model, the fluid senses a sudden temperature change at the entry where the dispersion is assumed to begin. This temperature change ($T_{in}^{-} - T_{in}^{+}$) is inversely proportional to the Péclet number for parabolic dispersion but in case of hyperbolic model the corresponding equation can be derived assuming an adiabatic nondispersive flow in the conduit before the entry to the apparatus. This gives an axial heat flux of zero before the entry. After the entry the temperature change can be calculated by carrying out an energy balance at the entry section as

$$F_q q_{x,in}^{+} + \dot{m}h_{in}^{-} = \dot{m}h_{in}^{-}. \tag{21}$$

Neglecting any pressure change of the real fluid in the inlet cross-section and choosing an appropriate mean value for c_p between T_{in}^{+} and T_{in}, the heat flux just after the entry can be derived as

$$q_{x,in}^{+} = w\rho c_p(T_{in}^{-} - T_{in}^{+}). \tag{22}$$

Using the third sound wave formulation for axial dispersive heat flux as per equation (3) after applying operator H to equation (22), yields

$$-\lambda\frac{\partial T_{in}^{+}}{\partial x} = w\rho c_p(T_{in}^{-} - T_{in}^{+}) + \frac{w\lambda}{C^2}\frac{D(T_{in}^{-} - T_{in}^{+})}{D\tau} \tag{23}$$

but $(DT_{in}^{-}/D\tau) = O$ since the channel is adiabatic and nondispersive before entry hence we get the final form of boundary condition

$$T_{in}^{-} - T_{in}^{+} = -\frac{\alpha}{w}\left(1 - \frac{w^2}{C^2}\right)\frac{\partial T_{in}^{+}}{\partial x} + \frac{\alpha}{C^2}\frac{\partial T_{in}^{+}}{\partial \tau}. \tag{24}$$

By nondimensionalising this can be reduced to

$$T_{in}^{-} - T_{in}^{+} = -\frac{1-M^2}{Pe}\frac{\partial T}{\partial \xi} + \frac{M^2}{Pe}\frac{\partial T}{\partial z}. \tag{25}$$

It is very easy to examine that as the dispersive mach number, $M \to O$ (which means the velocity of third sound wave approaches infinity) the boundary condition reduces to normal Danckwerts boundary condition. At the outlet the usual zero slope condition can be used.

1.5. Case 2, M > 1

The supersonic boundary condition with respect to third sound wave is somewhat different and complex. In the subsonic regime the disturbance comes first and then comes the fluid hence an observer moving with the fluid comes across the temperature drop at entry which has already taken place due to dispersion. In supersonic case the situation is analysed as follows.

The general flux equation (5) is valid for both Fourier and non-Fourier type dispersion law. The non Fourier law for dispersion can be obtained by using the hyperbolic law for dispersion and replacing $(\partial/\partial\tau)$ by $(D/D\tau)$. The substantial derivative is used because the changing temperature field due to dispersion comes to an observer who moves along with the fluid. This results in a flux equation

$$\dot{q}_x + \frac{a}{C^2}\frac{\partial \dot{q}_x}{\partial \tau} + \frac{aw}{C^2}\frac{\partial \dot{q}_x}{\partial x} = -\lambda\frac{\partial T}{\partial x}. \tag{26}$$

When the moving observer crosses the entry section he suddenly marks a change in the dispersion characteristic where the dispersion parameter λ attains a finite positive value from the previous value of 0 (no dispersion in the leading channel). Therefore, at $x = 0^-$, $\lambda = 0$ giving $\dot{q}_x = 0$ and at $x = 0^+$, $\lambda > 0$

$$\dot{q}_x + \frac{a}{C^2}\frac{D\dot{q}_x}{D\tau} = -\lambda\frac{\partial T}{\partial x}\bigg|_{x=0^+}. \tag{27}$$

So, in the space domain the heat flux shows a change as shown in Fig. (2). However, the hyperbolic law intro-

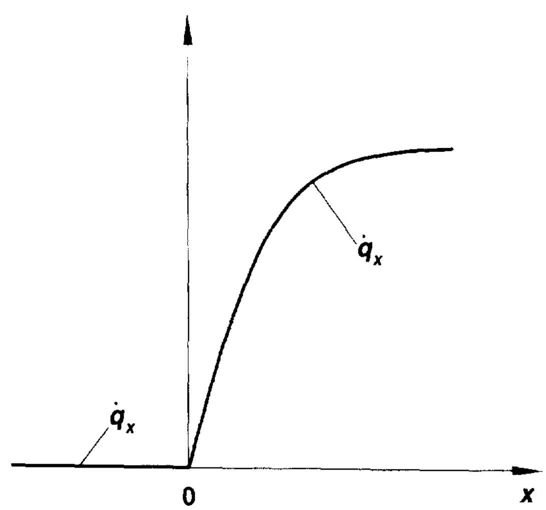

Fig. 2. Change of heat flux at the entry of the channel.

duces a delay in the heat flux and hence the flux does not come to the entry position giving still a zero flux at $x = 0$. This is analogous to an inductive resistance in an electrical circuit. This means at $x = 0^+$, $q_x = 0$ giving

$$\frac{a}{C^2}\frac{D\dot{q}_x}{D\tau} = -\lambda \left.\frac{\partial T}{\partial x}\right|_{x=0^-}. \tag{28}$$

Thus for a static observer at $x = 0$, always

$$q_x|_{x=0^+} = 0.$$

This makes the operator

$$\left.\frac{D}{D\tau}\right|_{x=0^+} = w\left.\frac{\partial}{\partial x}\right|_{x=0^+}$$

reducing equation (28) to

$$\left.\frac{\partial \dot{q}_x}{\partial x}\right|_{x=0^+} = -\frac{\lambda C^2}{aw}\frac{\partial T}{\partial x}. \tag{29}$$

Substitution of this flux value to the general flux equation (26) results

$$\left.\frac{\partial T}{\partial x}\right|_{x=0^+} = \frac{\frac{\alpha U}{F_q \rho c_p w}(T_w - T) - \frac{1}{w}\left.\frac{\partial T}{\partial \tau}\right|_{x=0^-}}{1 - \frac{C^2}{w^2}}. \tag{30}$$

This can be reduced to the dimensionless form

$$\left.\frac{\partial T}{\partial \xi}\right|_{\xi=0^+} = \frac{NTU(T_w - T) - \left.\frac{\partial T}{\partial z}\right|_{\xi=0}}{1 - \frac{1}{M^2}}. \tag{31}$$

In fact this can be stated as the first boundary condition at entry while the second being

$$T_{in} = T_{in}^-. \tag{32}$$

This means for the supersonic case two boundary conditions at entry and none at exit. This can be explained in the light that for $M > 1$ the hyperbolic law is a one way behaviour and hence there is no feedback of information from the exit to the entry unlike the subsonic case. Temperature profiles are sketched in Fig. (3) for both the boundary conditions.

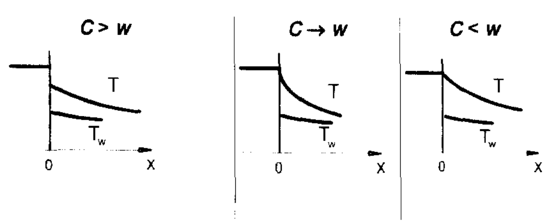

Fig. 3. The temperature sketch at entry for different ranges of third sound Mach number.

1.6. Exit conditions: A special consideration

The boundary condition at exit also seems to be influenced by dispersion but this is not required for the calculation of temperature profile for supersonic condition ($M > 1$) as explained in the preceding section. Whatever may be the dispersive mach number, an energy balance assuming no dispersion in the adiabatic channel taking the fluid axial heat flux due to dispersion, gives

$$w\rho c_p F_q (T_{out}^+ - T_{out}^-) = F_q \dot{q}_x|_{\xi=1^-}. \tag{33}$$

The calculation of $\dot{q}_x$ at exit can be done from the hyperbolic law for dispersion as in equation (3), giving

$$\dot{q}_x = \frac{-\lambda\, \partial T/\partial x}{1 + \frac{a}{C^2}\frac{D}{D\tau}}. \tag{34}$$

This can be expanded and approximated taking only the first two terms as

$$\dot{q}_x = -\lambda\frac{\partial T}{\partial x}\left[1 - \frac{a}{C^2}\frac{D}{D\tau}\right] \tag{35}$$

yielding

$$\dot{q}_x = -\lambda\frac{\partial T}{\partial x} + \frac{\lambda a}{C^2}\frac{\partial^2 T}{\partial x \partial \tau} + \frac{\lambda a w}{C^2}\frac{\partial^2 T}{\partial x^2}. \tag{36}$$

Substitution of this equation to equation (33) yields

$$T_{out}^+ - T_{out}^- = -\frac{\lambda}{w\rho c_p}\frac{\partial T}{\partial x} + \frac{\lambda a}{C^2 w \rho c_p}\frac{\partial^2 T}{\partial x \partial \tau} + \frac{\lambda a}{C^2 \rho c_p}\frac{\partial^2 T}{\partial x^2}. \tag{37}$$

This can be nondimensionalized to

$$T_{out}^+ - T_{out}^- = -\frac{1}{Pe}\frac{\partial T}{\partial \xi} + \frac{M^2}{Pe^2}\left(\frac{\partial^2 T}{\partial \xi \partial z} + \frac{\partial^2 T}{\partial \xi^2}\right). \tag{38}$$

This equation results in a temperature jump at the exit as shown in Fig. (4). Since in reality such a temperature jump at exit is not likely to be encountered, it requires a physically sound explanation.

The temperature jump at the exit is in reality no temperature change but the transfer from the spatial mean temperature at the end of the heat transfer surface (e.g. the arithmetic mean of all tubeside outlet temperatures in a tube bundle) to the adiabatic mixing temperature in the outlet cross-section.

The longitudinal temperature profile in the exchanger according to the dispersion model represents the spatial mean temperature which is decisive for the mean heat flux in a plug flow cross-section. This temperature is shown in Fig. (4) as a solid line. The dotted line represents the adiabatic mixing temperature which has to be used for plug flow in the one-dimensional energy equation.

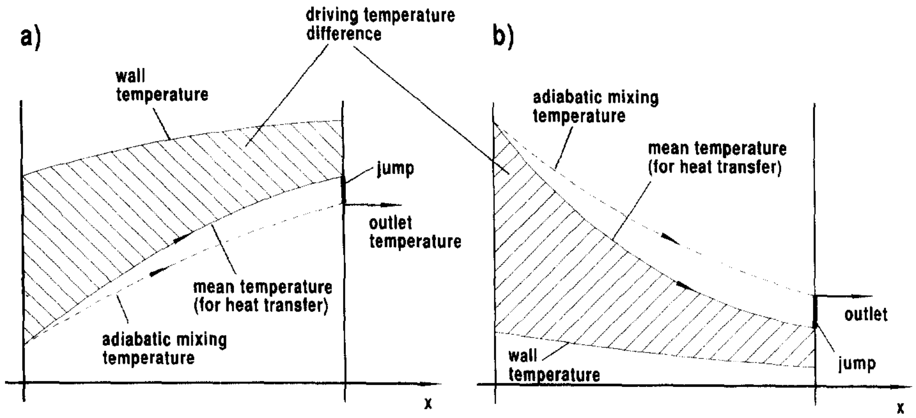

Fig. 4. The temperature jump at exit: (a) heated fluid; (b) cooled fluid.

But this temperature is not the driving force for heat transfer. In the outlet cross-section both temperatures are reunited because one mean velocity and one mean temperature can be assumed.

To illustrate the difference between these two temperatures let us consider two streams of fluid with velocities w_1 and w_2 flowing though two channels of width B_1 and B_2 respectively as shown in Fig. (5). The temperature change for similar thermal boundary conditions for the channel (e.g. constant wall heat flux) can be found to be

$$\Delta T_1 > \Delta T_2 \quad \text{for} \quad w_1 < w_2.$$

The spatial mean temperature at the end can be calculated as

$$\Delta T_m(B_1+B_2) = \Delta T_1 B_1 + \Delta T_2 B_2. \tag{39}$$

The adiabatic mixing temperature must take care of the velocities yielding

$$\Delta T_{ad}(B_1 w_1 + B_2 w_2) = \Delta T_1 B_1 w_1 + \Delta T_2 B_2 w_2. \tag{40}$$

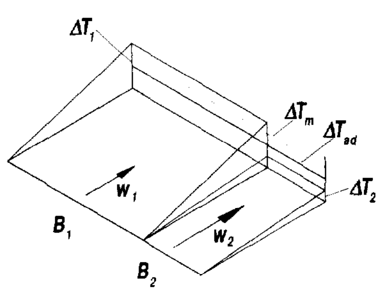

Fig. 5. Fluid flowing through two adjacent channels with different temperature and velocity—demonstration of the difference between mean and adiabatic mixing temperature.

Hence the difference between spatial mean temperature and adiabatic mixing temperature can be derived as

$$\Delta T_m - \Delta T_{ad} = (\Delta T_1 - \Delta T_2) \times \left(\frac{B_1}{B_1+B_2} - \frac{B_1}{B_1 + B_2 \cdot \frac{w_2}{w_1}} \right). \tag{41}$$

Since both the terms in the right hend side of this equation have the same sign hence it can be inferred that the spatial mean temperature changes faster than the adiabatic mixing temperature. The case is changed for a fluid losing heat. The sketch of the two temperatures in this case is shown in Fig. 4(b).

2. Examples, results and discussion

Based on the theoretical foundation of the 'third sound wave' formulation the results of some specific examples have been calculated. It has been applied to simple cases such as a laminar flow though tube as well as complex flow patterns such as a multiple channel plate and frame heat exchanger.

2.1. Example 1: Heat transfer due to laminar flow in a tube

This simple case is chosen to exhibit the strength of the sound wave formulation of axial dispersion. Here the flow is assumed to take place in an adiabatic tube. The conduction in the tube wall is neglected since the tube is considered to be thin. Fluid properties such as specific heat and dispersion coefficients and also the third sound mach number are considered to be constant. Under such assumptions the equation (17) is valid for the fluid.

The wall temperature is described by

$$B'\frac{\partial \vartheta_w}{\partial z} = NTU(\vartheta - \vartheta_w) \tag{42}$$

where

$$B' = \frac{\delta_w \rho_w c_{pw}}{2R\rho c_p}$$

and

$$NTU = \frac{2\alpha L}{R\rho c_p w}. \tag{43}$$

Equations (17) and (18) with $\gamma_w = 0$ can be solved by using the boundary conditions. Calculations have been done for subsonic case using boundary condition of equation (25). The solution is derived by using Laplace transformation in the form:

$$\bar{\vartheta} = \frac{[(A-D)\,e^{-D(1-\xi)} - (A+D)\,e^{D(1-\xi)}]\,e^{A\xi}\,PeF(s)}{(Pe+M^2 s)\left\{(A-D)\,e^{-D}\left[1-\dfrac{1-M^2}{Pe+sM^2}(A+D)\right] - (A+D)e^{D}\left[1-\dfrac{1-M^2}{Pe+sM^2}(A-D)\right]\right\}}$$

where

$$A = \frac{Pe}{2(1-M^2)}\left[1+\frac{sM^2}{Pe}\left(2+\frac{B'NTU}{sB'+NTU}\right)\right]$$

$$D = \sqrt{A^2 + \frac{sPe}{1-M^2}\left(1+\frac{sM^2}{Pe}\right)\left(1+\frac{NTUB'}{sB'+NTU}\right)}. \tag{44}$$

This can be inverse transformed into time domain using Crump's [24] algorithm which uses a complex Fourier series approximation of the function.

The results have been calculated for different values of the dispersive mach number. For comparison the exact solution of a two dimensional fully developed flow with parabolic velocity profile has been considered. Figure (6) shows that the $M = 1$ solution agrees well when the radial heat conduction characterised by $\eta^2 Pe_L$ is small and consequently the dispersive Péclet number Pe is large. For larger dispersion (i.e., smaller Pe) $\eta^2 Pe_L$ is larger as given in [25]. In Fig. 7 it is observed that for a value of 10 for $\eta^2 Pe_L$ the dispersive Péclet number of 8.5 and a third sound mach number of $M = 1$ gives more satisfactory result than with $M = 0$. This indicates one more important inference that the guessed paradigm of third sound wave propagation velocity of the order of fluid velocity is perfectly justified and under the action of strong dispersion it is important to consider the third sound wave in fluid.

2.2. *Example 2: Infinitely long adiabatic channel*

The second example chosen to suggest the third sound effect is an adiabatic channel of infinite length initially at constant temperature $\vartheta = 0$. At $x = 0$ suddenly the temperature is changed to $\vartheta = 1$ (at time, $z = 0$). The channel under consideration is of length L which is the part of an infinitely long one. The equation for such a channel without heat transfer with the wall is

$$M^2\left(\frac{\partial^2\vartheta}{\partial z^2} + 2\frac{\partial^2\vartheta}{\partial z\,\partial\xi} + \frac{\partial^2\vartheta}{\partial\xi^2}\right) + Pe\left(\frac{\partial\vartheta}{\partial z} + \frac{\partial\vartheta}{\partial\xi}\right) = \frac{\partial^2\vartheta}{\partial\xi^2}. \tag{45}$$

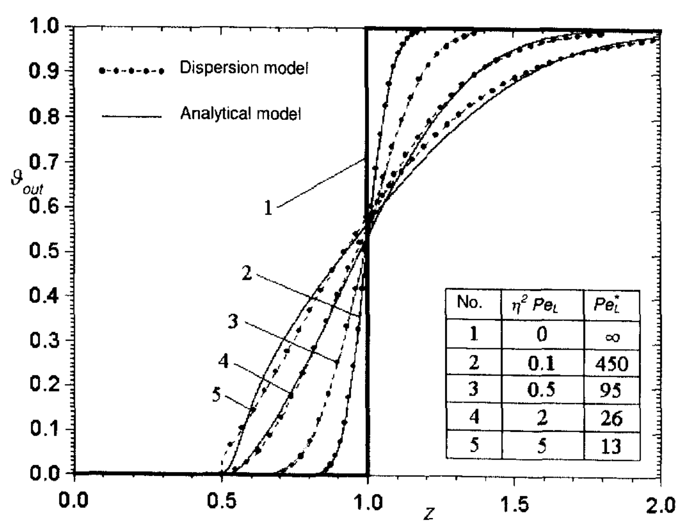

Fig. 6. Transient temperature response of a fluid flowing through an adiabatic tube. Comparison with hyperbolic dispersion ($M = 1$).

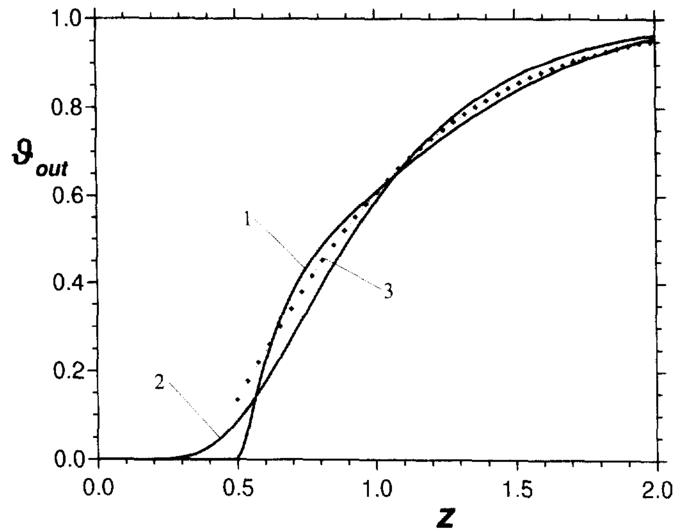

Fig. 7. Transient response for flow through a tube with large dispersion ($\eta^2 Pe_L = 10$). 1. analytical model, 2. parabolic dispersion model ($M = 0$, $Pe = 8.5$) and 3. third sound model ($M = 1$, $Pe = 8.5$).

The boundary and initial conditions are

$$\begin{aligned} \vartheta(z>0,\xi\to\infty) &= 0\\ \vartheta(z>0,\xi\to-\infty) &= 1\\ \vartheta(z=0,\xi\geqslant 0) &= 0\\ \vartheta(z>0,\xi<0) &= 1 \end{aligned} \tag{46}$$

and

$$\left.\frac{\partial\vartheta}{\partial z}\right|_{z=0,\xi} = 0.$$

The equation (45) which is normally named as telegraphic equation can be solved analytically to give the temperature at exit, as

$$\vartheta_{out}(z) = e^{(x_0-y_0)}\left\{\frac{1}{2}(\phi(x_0)+\phi(y_0)) + \int_{x_0}^{y_0}\phi(\varsigma)\sum_{k=0}^{\infty}\frac{[-(\varsigma-x_0)(\varsigma-y_0)]^k}{(k!)^2}\,d\varsigma + \frac{1}{2}(y_0-x_0)\int_{x_0}^{y_0}\phi(\varsigma)\sum_{k=0}^{\infty}\frac{[-(\varsigma-x_0)(\varsigma-y_0)]^k}{k!(k+1)!}\,d\varsigma\right\} \tag{47}$$

where

$$\begin{aligned} x_0 &= -\frac{Pe}{4M}\left(1-z+\frac{z}{M}\right)\\ y_0 &= -\frac{Pe}{4M}\left(1-z-\frac{z}{M}\right) \end{aligned} \tag{48}$$

and

$$\phi(s) = \begin{cases} 0 & \text{for } \varsigma \leqslant 0\\ 1 & \text{for } \varsigma > 0 \end{cases}.$$

The solutions are plotted in Figs. 8 and 9 for large ($Pe = 5$) and small ($Pe = 100$) magnitudes of dispersion. It is important to note that the temperature response of the parabolic dispersion model ($M = 0$), where the third sound velocity is infinitely large, shows a smooth and gradual rise while for a finite third sound velocity it suddenly rises to a finite value. Not only at the time of beginning but also at the end the temperature suddenly rises to its final value in the case of a supersonic flow. This clearly demonstrates a wave like propagation of the dispersion.

The nature and magnitude of the temperature jump depends on the dispersive Péclet number and mach numbers as evident from the figures. The first jump can be calculated to occur at $Y_0 = 0$, giving

$$z = \frac{M}{M+1}. \tag{49}$$

The magnitude of the jump being

$$\vartheta = \frac{1}{2}\exp\left[-\frac{Pe}{2M(1+M)}\right].$$

The second jump for $M > 1$ occurs at $x_0 = 0$, giving

$$z = \frac{M}{M-1}. \tag{50}$$

Even though this case is a highly theoretical one it brings out the important features of a dispersion wave in

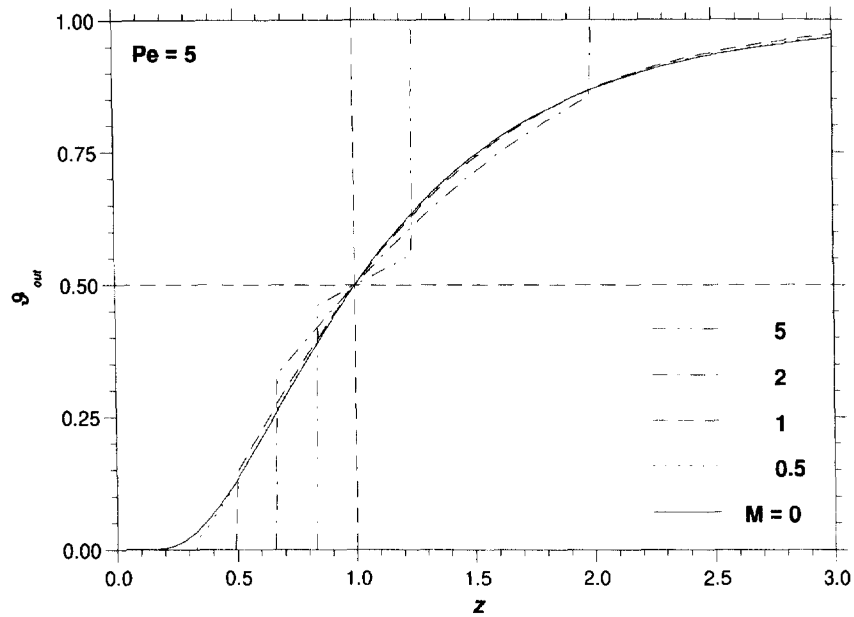

Fig. 8. Temperature response of an infinitely long tube with large dispersion.

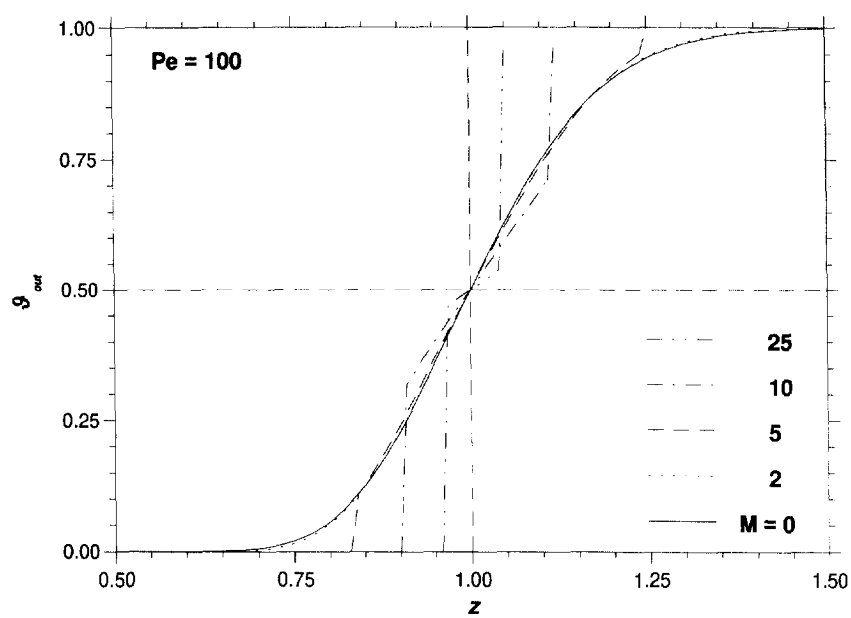

Fig. 9. Temperature response of an infinitely long tube with near plug flow (small dispersion).

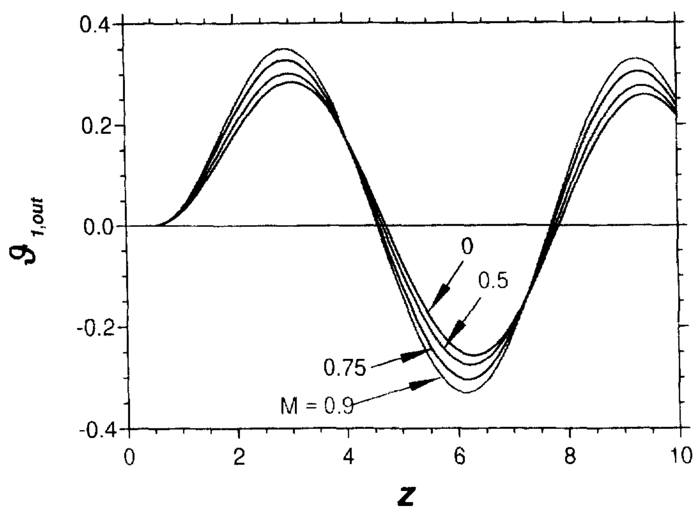

Fig. 10. Effect of third sound mach number on sinusoidal temperature response of a plate heat exchanger.

the form of third sound which can be studied by future investigators.

2.3. *Example 3: Heat transfer in a plate heat exchanger*

It is worthwhile to reproduce the features of hyperbolic dispersion wave in a complete heat transfer apparatus as shown in ref. [21]. *U* and *Z* type plate heat exchangers have been simulated here considering a hyperbolic dispersion in fluid taking the phase lag effect at the channel entry and exit into consideration. The detailed mathematical formulation is presented elsewhere [21]. In responses due to temperature oscillation and step change at entry has been calculated by the model. For sinusoidal [Fig. (10)] response the third sound mach number is found to influence the result considerably which can probably be used to explain the mismatch between the experimental and computed results observed while using a parabolic dispersion with infinite propagation velocity of third sound wave [8]. More extensive experiments are underway to assess the heat transfer in a complete apparatus with higher level of confidence.

3. Summary and conclusion

An exhaustive theoretical foundation has been laid in this paper for the recent developments in the studies regarding a wave like propagation of axial dispersion. The wave can be named as the third sound wave in fluid which results from disturbances which affect the so called 'plug flow'. The fundamental equations have been derived from first principles and they have been reduced to a usable form by reasoning the physical aspects. It has been found that theoretically the subsonic and supersonic flows brings out two different types of behaviours at the boundary which has been closely examined with physical explanation. Three different examples have been presented: first, a simple practical flow condition; second, a theoretical behaviour and third the application to a heat transfer apparatus. The examples demonstrate the validity, nature, characteristic and possibility of application to heat transfer equipment.

Standing on the foundation of the theory presented here it is expected that the future investigations will concentrate on

(i) Experimental determination of parameters like dispersive Péclet (*Pe*) and Mach (*M*) numbers.

(ii) Indirect determination *Pe* and *M* from complete numerical simulation of heat transfer equipment and flow geometries.

(iii) Suggesting proper modelling of *Pe* and *M* in the form of algebraic or differential equations, which can be termed as 'dispersion modelling'.

(iv) From the calculated parameters as done in the previous suggestions [(i), (ii) and (iii)] simulate a total heat transfer equipment and carry out experiments to validate it.

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